THE QUESTION | CASES ON A LINE

There are three points P, Q and R on a straight line such that PQ : QR = 3 : 5. If n is the number of possible values of PQ : PR, then what is n equal to?

Ratios on a Line | Case Analysis | Hidden Trap: assuming R lies beyond Q

(a) 1
Takes only one position of R. R can be on either side of Q.

(b) 2 ✓
R beyond Q gives 3 : 8. R on the side of P gives 3 : 2.

(c) 3
There is no third position. Q splits the line into two sides only.

(d) 4
There is no third or fourth position. Only two sides of Q exist.

CRACK THE HIDDEN RULE

Three points on a line with a fixed gap ratio can sit in more than one order. Take PQ = 3 and QR = 5 and ask where R can be relative to Q.

R is either on the side of Q away from P, or on the same side as P.

PR = PQ + QR = 8, or PR = QR − PQ = 2

R BEYOND Q	R TOWARDS P	n
PR = 8, ratio 3 : 8	PR = 2, ratio 3 : 2	2

WORKED IN 3 CLEAN STEPS

1 Fix lengths
Let PQ = 3 and QR = 5.

2 Case 1: R on the far side of Q
PR = 3 + 5 = 8, so PQ : PR = 3 : 8.

3 Case 2: R on P's side of Q
PR = 5 − 3 = 2 (R is beyond P), so PQ : PR = 3 : 2. Two values, so n = 2.

ANSWER: (b) 2

THE 3-SECOND SHORTCUT

Two points fix a line. The third can lie on either side of the middle one. So PR is either the sum or the difference of the two given lengths, giving exactly two ratios.

sum or difference | two sides of Q | never assume the order

→ Both ratios, 3 : 8 and 3 : 2, are different, so both count.
→ Check: PQ = 3 and QR = 5 hold in both pictures.

WHY ASPIRANTS LOSE MARKS HERE

1. **Taking only P, Q, R in that order.** The question does not fix the order.
2. **Counting 4 cases from permutations.** On a line only two sides of Q exist.
3. **Forgetting R can lie beyond P.** QR is longer than PQ, so R passes P.

ATTEMPT	FLAG	SKIP
~30 sec	if a sketch helps	never
Two cases, two ratios.	Draw P, Q and both positions of R.	Very quick.

THE ONE-LINE TAKEAWAY

When order is not fixed on a line, test the sum and the difference of lengths.

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