THE QUESTION | REPUNIT x 7

When a certain number is multiplied by 7, the product entirely comprises ones only (1111...). What is the smallest such number?

Divisibility by 7 | Repunits | Hidden Trap: three options end in 3, so the last digit alone does not decide

(a) 15718
7 x 15718 = 110026. The last digit is 6, not 1.

(b) 15723
7 x 15723 = 110061. Ends in 1 but is not all ones.

(c) 15783
7 x 15783 = 110481. Ends in 1 but is not all ones.

(d) 15873 ✓
7 x 15873 = 111111. Six ones, the first repunit divisible by 7.

CRACK THE HIDDEN RULE

A product made only of ones is a repunit: 1, 11, 111 and so on. The smallest multiplier is that repunit divided by 7, so find the first repunit that 7 divides.

Build repunits one at a time and track the remainder: the next remainder is (previous x 10 + 1) mod 7.

Remainder chain mod 7: 1, 4, 6, 5, 2, 0. Six ones divide by 7.

FIRST REPUNIT	DIVIDE	ANSWER
111111 (six ones)	111111 / 7	15873

WORKED IN 3 CLEAN STEPS

1. **Find the first repunit divisible by 7**
   Remainders of 1, 11, 111, 1111, 11111 are 1, 4, 6, 5, 2. The next, 111111, leaves 0.

2. **Divide**
   111111 / 7 = 15873.

3. **Match the option**
   Only (d) is 15873. Check: 7 x 15873 = 111111.
   ANSWER: (d) 15873

THE 3-SECOND SHORTCUT

Do not divide long repunits. Use the remainder chain: take the previous remainder, multiply by 10, add 1, then reduce mod 7. The chain hits 0 at six ones, and 111111 / 7 is the answer.

repunit = 1, 11, 111... | remainder x 10 + 1 | stop when remainder is 0

→ Option (a) is out at once, since 7 x ...8 ends in 6.
→ The other three need the full product or the chain to separate.

WHY ASPIRANTS LOSE MARKS HERE

1. **Stopping at the last digit.** Three options end in 3, so the last digit cannot decide.
2. **Testing 1111 or 11111.** They leave remainders 5 and 2, not 0.
3. **Multiplying every option in full.** The remainder chain finds the repunit in one line.

ATTEMPT	FLAG	SKIP
~45 sec	if long division slows you	rarely
Run the chain once and divide.	Multiply 15873 x 7 to confirm instead.	Mechanical, so a scorer.

THE ONE-LINE TAKEAWAY

For all-ones products, find the first repunit the divisor divides, then divide.

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